**RSTDPT (Adiabatic)**

**Example: Delta function**

Let’s specialize to a delta function potential. Turns out we can solve this one exactly, like we did for the constant perturbation, but this will be way easier.



We’d like to calculate the time-dependence of the wavefunction. We’ll start with:



where



And we’ll observe that the on-shell T-matrix is simply related to the off-shell T-matrix.



(and also, while V is constant in the off-shell T-matrix, it is not, for the on-shell shell T-matrix) Recall from the constant perturbation file that the off-shell T-matrix looks like this:



We’d like to solve for T. The best way to solve for T is via a recursion relation (trying to sum infinite series isn’t typically the way we want to proceed). Well, from that infinite series, we see that we can write,



And now let’s put this in a basis. We’ll dot both sides with <k| and |k´>, and insert resolutions of identity as needed,



Now the nice thing about the δ potential is that



So we have:



Pondering this equation a bit, we might observe that on the right hand side, the k1 index of Tk1k´ is integrated over, meaning the result of the integral will only depend on the k´ index. Thus considering Tkk´(ω) is equal to this thing, that means that Tkk´(ω) can at most depend on its k´ index alone. Given this, we can eliminate the first index of T from its argument, and write,



which makes it now trivial to solve for T,



But we’ll now observe that the RHS doesn’t even depend on k´. So really, T doesn’t have any index dependence at all – that’s what’s nice about the delta function potential. So we have, changing variable of integration from k1 -> k:



Let’s now do this integral,



If ω > 0, we can factor the denominator, and close the contour, up, say, to get:



And if ω < 0, then we do similarly, and close the contour up, say,



If we agree that √(-|ω|) = i√ω, then we can combine these two results to simply say,



And now we can get our on-shell T-matrix.



Awesome. Now we’ll fill this into our wavefunction expansion.



Let’s put this in the x basis. And well, we can say,



Filling in our T-matrix, we have:



So we want to evaluate this integral,



I’ll fill in ψk(x) = Aeikx. Then we have:



For positive x, we must close the contour up, and for negative x, down. Proceeding,



which simplifies some to,



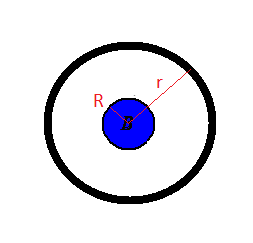
So our wavefunction is:



If we compare to the scattering folder, we’ll observe that the θ(-x) term is the incident and reflected wave, while the θ(x) is the transmitted wave!

**Example. Time-dependent perturbation of increasing magnetic flux through particle on a ring situation**

In the time-independent file, we solved for the eigenstates of a particle on a ring enclosing magnetic flux.



The time-dependent problem would be:



where we worked H down to:



where ΦB = BπR2. And we found eigenfunctions/energies to be:



I guess I’ll just point out that if we slowly increase B, then the change in energy would be:



where Mℓ is the magnetization of that level. And this is consistent with the formula for work done on a magnetic moment in a magnetic field (see EM folder/Insulator Energy). This is kind of interesting in that the work done, from a classical perspective is done by the electric field induced by the changing magnetic field. But in our quantum problem we do not explicitly talk about any induced electric field. Of course we *did* define M to be ∂E/∂B, so maybe we shouldn’t be too surprised. But maybe this is one reason why we must define it so.